Aircraft Design Optimization with Uncertainties

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Let’s Design a Plastic Chair

- Problem statement:
  - Design a chair that can hold a 100 kg load

- Choose a material
  - Polypropelene
  - Elastic modulus = 1.5 GPa
  - Yield strength 35 MPa
- Design the structure
  - Legs, ribs, seat, back
  - Material thickness, dimensions
  - Cross sections
- Run numerical simulations
- On the paper the chair can hold the load
- Manufacture it

Now Let's Test It

- Load the chair with a 100 kg mass

but why?

- **Our world is not deterministic!**

- Material is not perfect
  - Yield stress 34.684 ± 0.1234 MPa
- Manufacturing has tolerances $\pm 2\%$
- Test load can be 99.5 – 100.5 kg

How can we design a chair that will not break?

- **Engineers know the easy solution**

- **Apply Safety factors**
  - Design for 150 kg load, instead of 100
  - And/or Assume the $\sigma_{yield} = 30$ MPa, instead of 35
  - And/or Round off the wall thickness to a higher value. 3.28 mm -> 3.5

- This approach is simple, but inefficient and not precise
  - The safety factor lumps up all the uncertainties
  - It provides no insights which uncertainty matters the most

Two Types of Uncertainties

- **Epistemic**: due to lack of knowledge
  - Incomplete information or lack of understanding
  - **Can be reduced** with better models, accurate measurement, or data

- **Aleatory**: inherent variability
  - Comes from natural randomness in the system or environment
  - **Cannot be reduced**, only managed

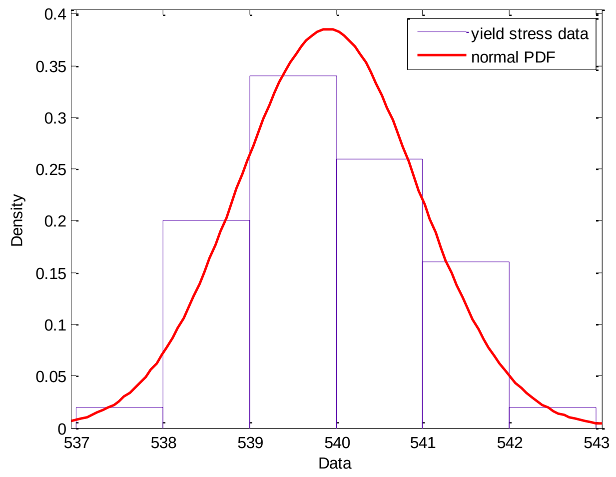
- Material properties due to inaccurate measurement
- FEM/CFD errors due to mesh resolution
- Uncalibrated sensors
- Low-fidelity analysis models

- Variability in raw material properties
- Environmental conditions (turbulence, weather)
- Load variations (traffic on bridges, vehicle payload)
- Random electrical noise
- Human/operator related variability

Probabilistic Approach to Uncertainty Modeling

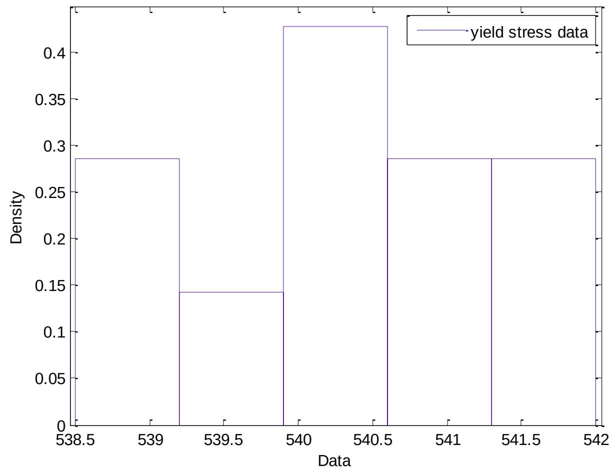
Yield Stress Data (MPa, 50 samples)				
540.89	539.90	539.14	538.91	539.38
538.85	539.76	540.08	540.03	540.75
538.93	540.32	538.79	540.55	539.81
539.19	540.31	538.89	541.10	540.89
537.06	539.14	539.99	541.54	539.24
541.44	539.97	541.53	540.09	538.60
540.33	539.84	539.23	538.51	538.58
539.25	540.63	540.37	539.26	540.49
541.37	541.09	539.77	538.94	539.82
538.29	541.11	541.12	542.35	539.80

Estimate PDF



Good PDF estimate not possible with only 10 samples

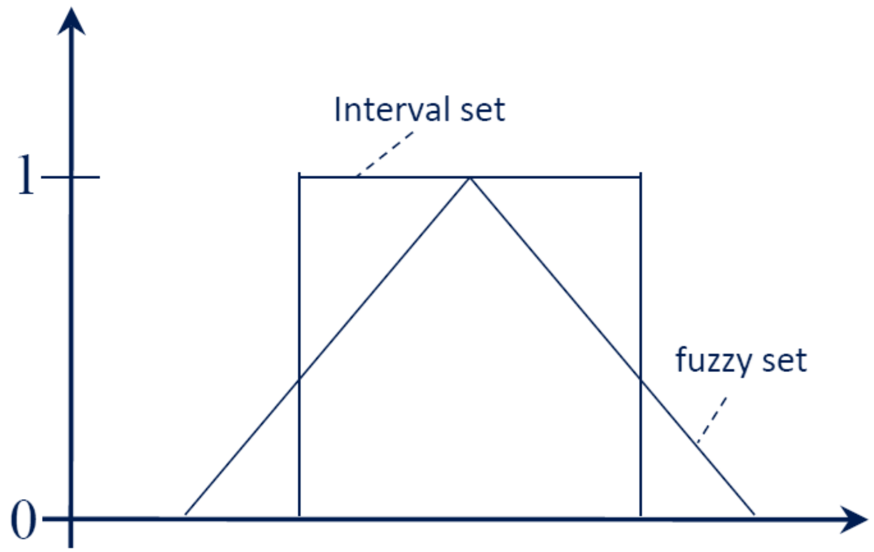
Yield Stress Data (MPa, 10 samples)				
541.42	540.29	540.20	541.59	539.20
540.70	540.84	539.76	540.22	538.83



Non-Probabilistic Approach

- Intervals
  - Considers the interval between the highest and lowest observation
  - Degree of Membership is either 0 or 1

- Fuzzy Numbers
  - Considers other degrees of membership according to a membership function

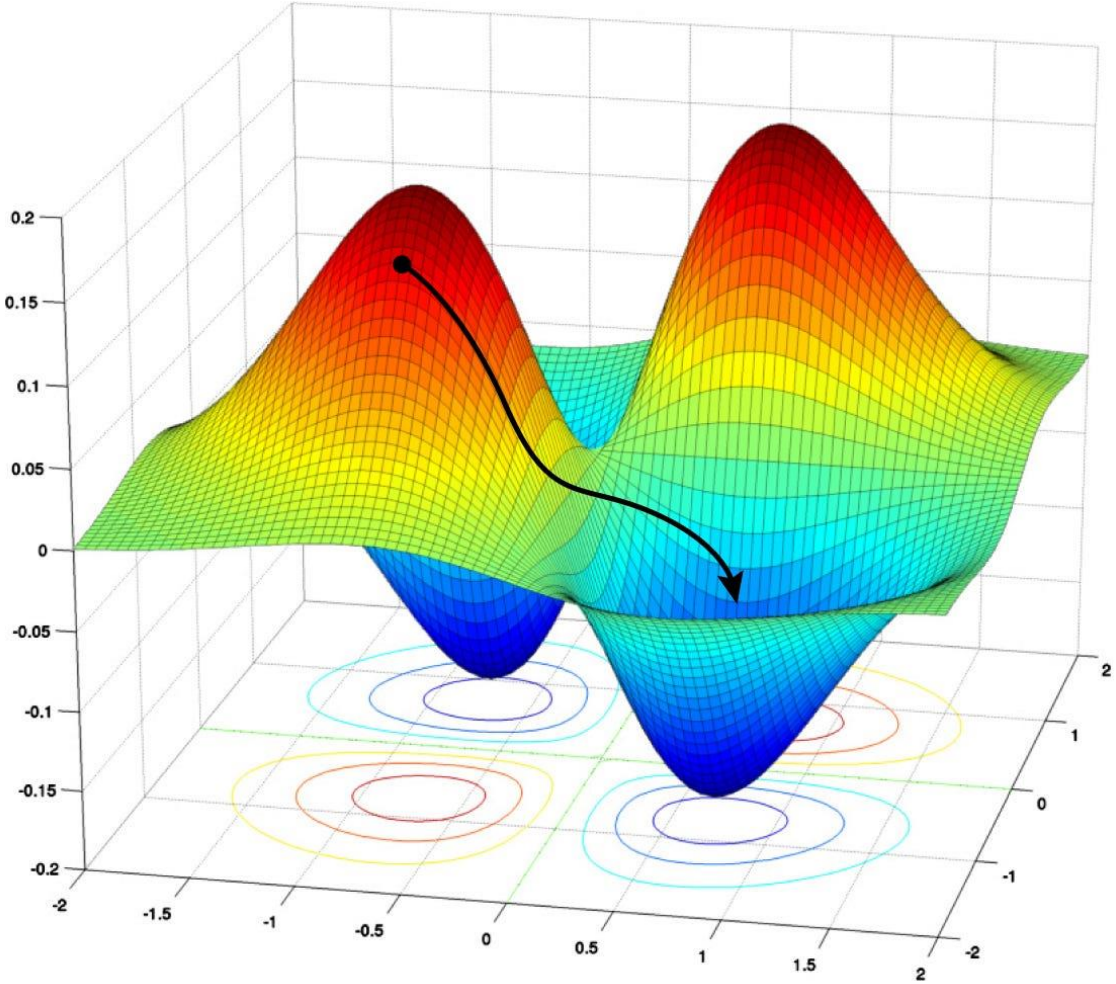


Deterministic Optimization Formulation

$$
\begin{aligned}
&\text{minimize} && f(x) \\
&\text{subject to} && h_i(x) = 0, \quad i = 1, \ldots, m_1 \\
& && g_j(x) \leq 0, \quad j = 1, \ldots, m_2 \\
&\text{and} && x \in X \subseteq R^n
\end{aligned}
$$

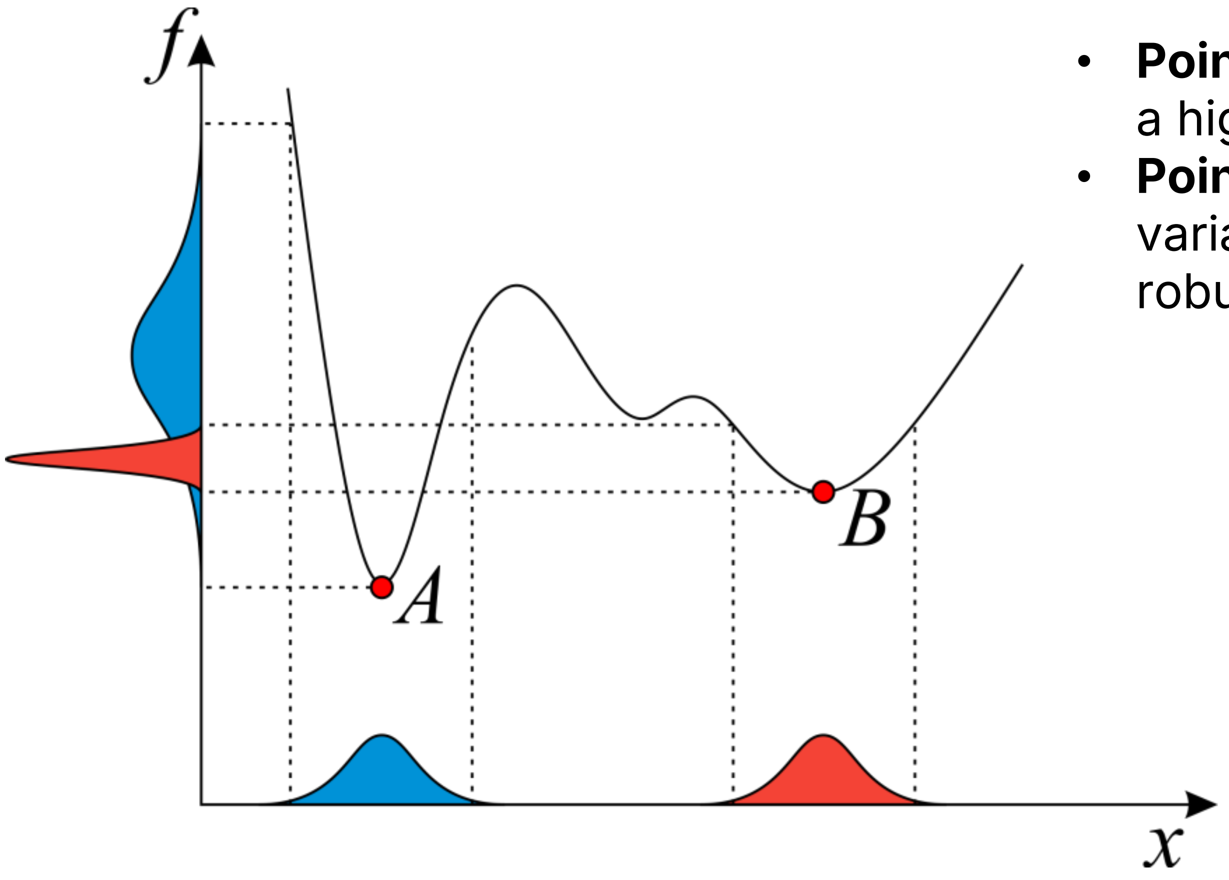
where

- x is a vector of *n* real-valued design variables $x_1, x_2, \ldots, x_n$
- $f(x)$ is the **objective function**
- $h_i(x)$ are m_1 **equality constraints**
- $g_j(x)$ are m_2 **inequality constraints**



Robust Design Optimization (RDO)

- Used to reduce the variability of objective function by minimizing the effects of uncertainty then removing the source of noise or uncertainty parameter effects



- **Point A** has a better value of objective but a higher variance
- **Point B** has a worse objective but a lower variance thus a more trustworthy and robust result

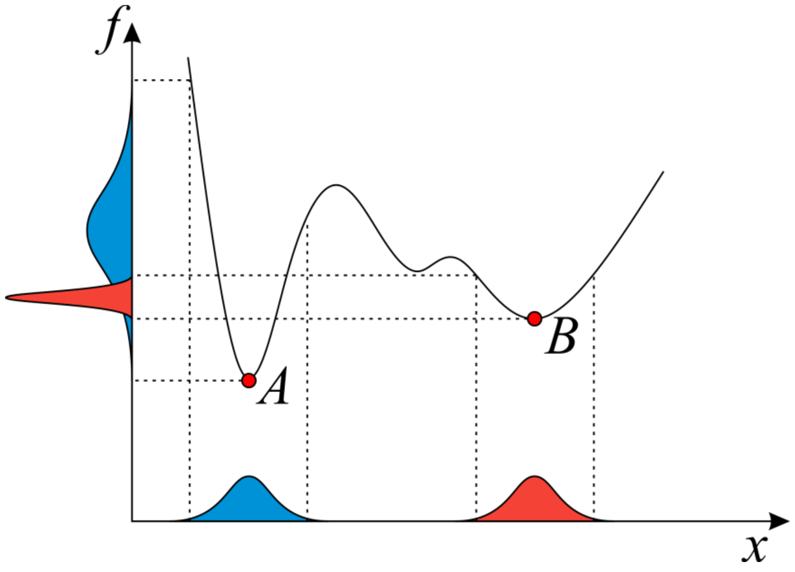
Optimization Formulation for RDO

- Minimize

$$z(d,x,p) = \mu_{\hat{f}} + \sigma_{\hat{f}}^2$$

- Subject to

$$g_i(d,\mu_x,\mu_p) \leq 0, i = 1,\ldots,n_c$$



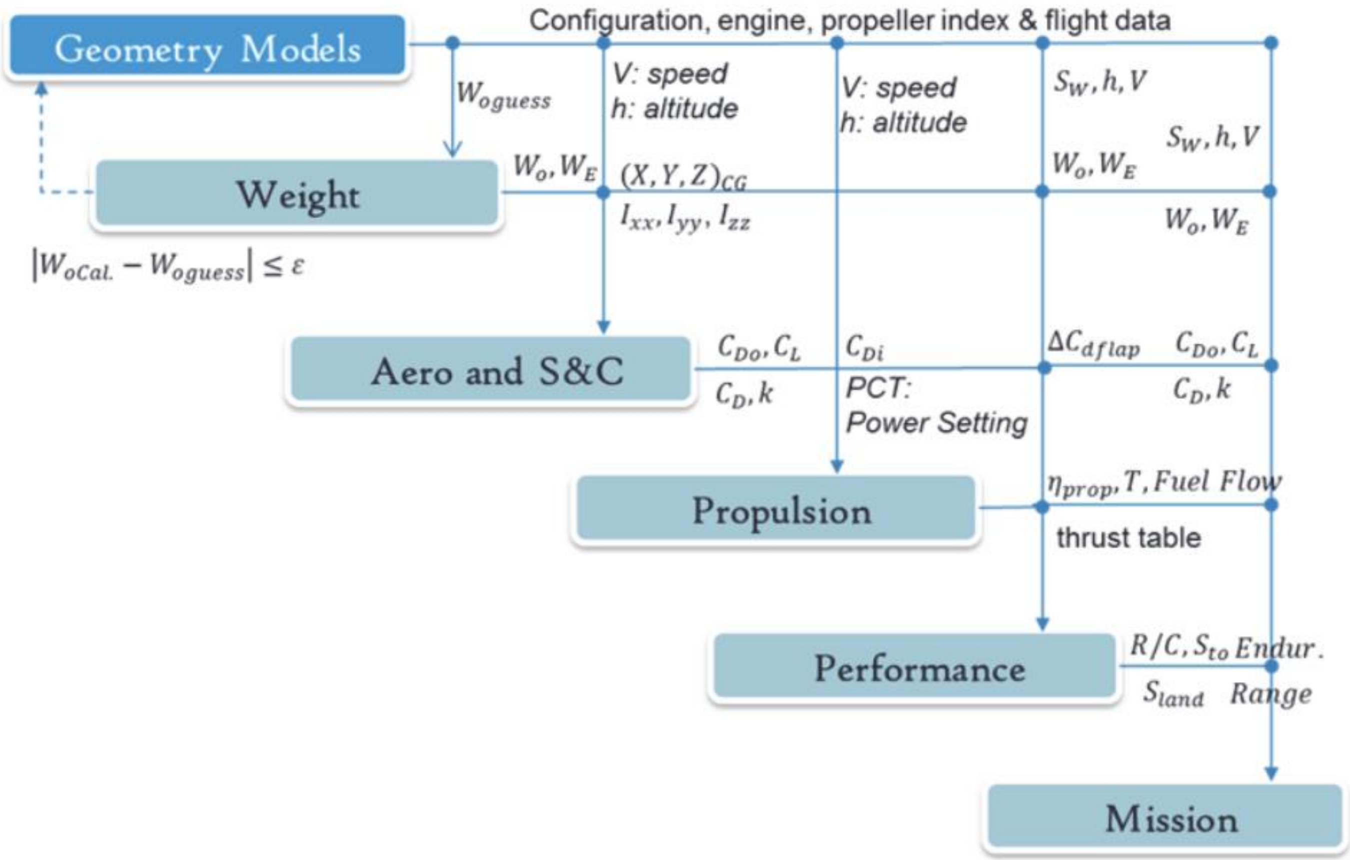
- Mean and the variance are usually normalized
- Variance of $f(x)$ is approximated using the first derivative

$$\mu_{\hat{f}} = f(d,\mu_x,\mu_p) \cdot \frac{1}{T_{\mu_f}} \qquad \sigma_f^2 \cong \sum_{i=1}^{nv} \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_{x_i}^2 + \sum_{i=1}^{np} \left(\frac{\partial f}{\partial p_i}\right)^2 \sigma_{p_i}^2$$

$$\sigma_{\hat{f}}^2 = \sigma_f^2 \cdot \frac{1}{T_{\sigma_f}^2}$$

RDO for MALE UAV Design

Predator A
Medium Altitude Long Endurance UAV



[1] N. V. Nguyen, J.-W. Lee, Y.-D. Lee, and H.-U. Park, "A multidisciplinary robust optimisation framework for UAV conceptual design," *The Aeronautical Journal*, vol. 118, no. 1200, pp. 123–142, Feb. 2014

Formulation

Objective

- Maximize flight endurance
- Minimize variance of the endurance

Uncertain parameters

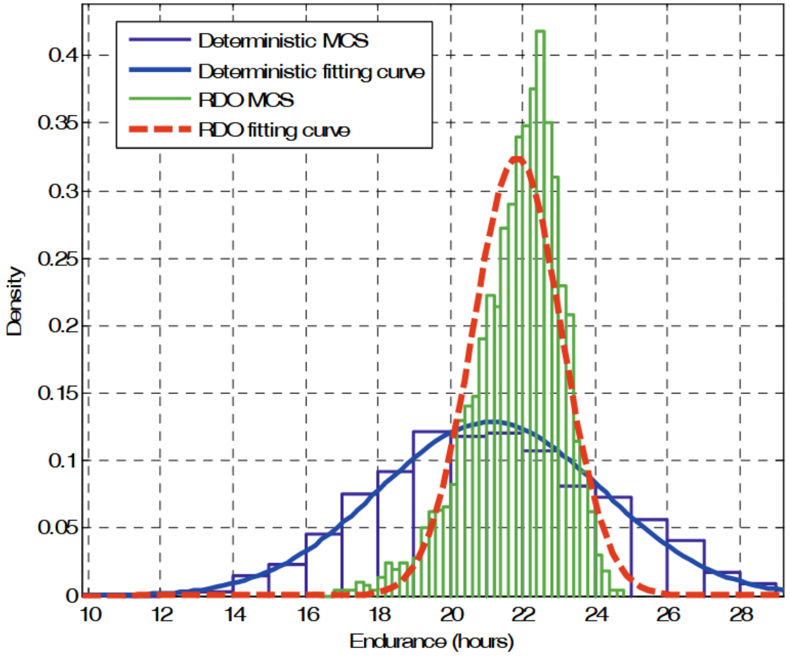
- Cruise altitude
- Cruise velocity

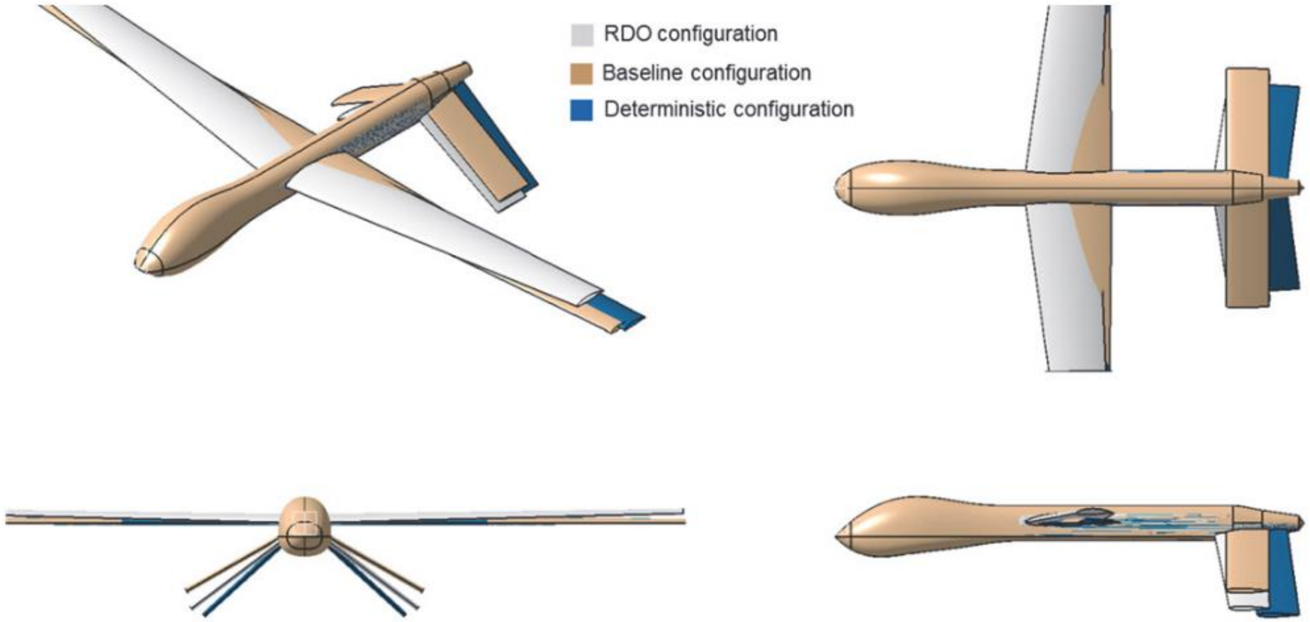
	Baseline	Lower Bounds	Upper Bounds	Unit
Wing span	14·8	10	20	m
Wing root chord	1·24	1·0	1·4	m
Wing tip chord	0·5	0·3	0·7	m
Wing sweep	5	0	10	deg
Wing dihedral	0	0	5	deg
Wing X location	3·59	3	4	m
HT span	4	3·5	4·5	m
HT root chord	0·742	0·4	1	m
HT tip chord	0·742	0·4	1	m
HT sweep	0	0	10	deg
HT X location	6·82	5·8	7·8	m
VT span	1·14	0·7	1·5	m
VT tip chord	0·742	0·4	1	m
VT root chord	0·742	0·4	1	m
VT LE sweep	0	0	60	deg
VT X location	6·82	5·8	7·8	m
V_{design}	42	27·78 (V_{stall})	64	ms^{-1}
h	3,000	2,000	4,000	m

Constraints	Description	Discipline
G(1,2)	Static margin: $0{\cdot}05 \leq SM \leq 0{\cdot}2$	S&C (@V_{design})
G(3)	Take-off field length ≤ 700 (m) (2,300ft)	Perf (50ft) (WOT)
G(4)	Lateral stability derivative: $C_{l\beta} \leq -0{\cdot}03$	S&C (@V_{design})
G(5)	Gross weight: MTOW ≤ 1,020kg	Weight
G(6)	Pitching moment der· $C_{m\alpha} \leq 0$	S&C (@V_{design})
G(7)	Landing distance ≤ 518m 1,700ft	Perf·
G(8)	Wing weight: Wwing ≤ $W_{baseline}$ (kg)	Weight
G(9)	Lift over drag ratio: $L/D \geq L/D_{baseline}$	Aeros (@V_{design})
G(10)	Wing taper ≥ 0·4	Geometry
G(11)	Take-off ground roll ≤ 438m (1,440ft)	Perf (@WOT)
G(12)	Maximum speed (V_{max}) ≥ 60·3ms^{-1}	Perf (@WOT)
G(13)	Stall speed (V_{stall}) ≤ 27·8ms^{-1}	Perf (Clean)
G(14)	Service ceiling ≥ 25,000ft	Perf (@WOT)
G(15,16)	Directional derivatives coefficient $0{\cdot}08 \leq C_{n\beta} \leq 0{\cdot}28$	S&C (@V_{design})
G(17)	Empty weight: We ≤ We_Baseline(kg)	Weight

Result of the Robust Design Optimization

	Deterministic	Robust
Mean endurance, hours	21.11	21.84
Endurance variance, hours	9.57	1.51
Probability $P(E \geq 21\ h)$	50.4%	78.4%





Managing the Constraints

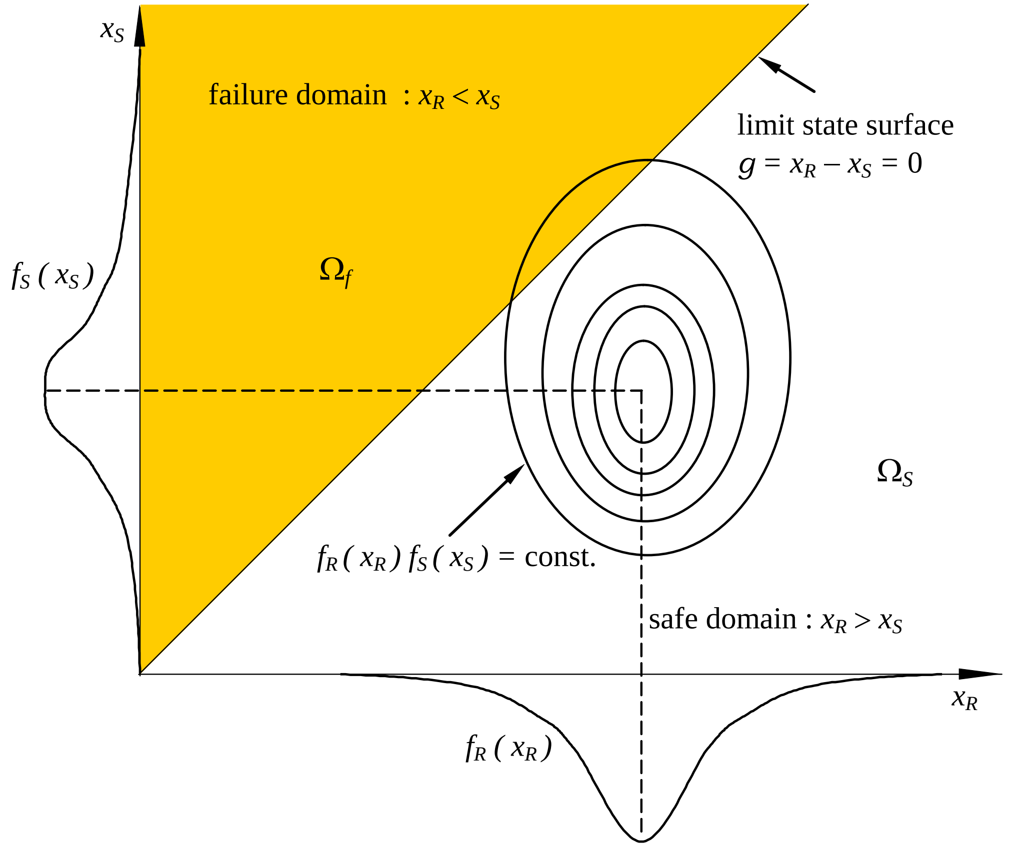
- Reliability based Design Optimization (RBDO)

$$\min f(\bar{x}, \bar{p}, y(\bar{x}, \bar{p}))$$

$$\text{where } i = 1, \ldots, N_{cons}$$

$$\text{s.t. } P\left[g_i(x, p, y(x, p)) > 0\right] \leq P_f$$

$$x_l \leq x \leq x_u$$

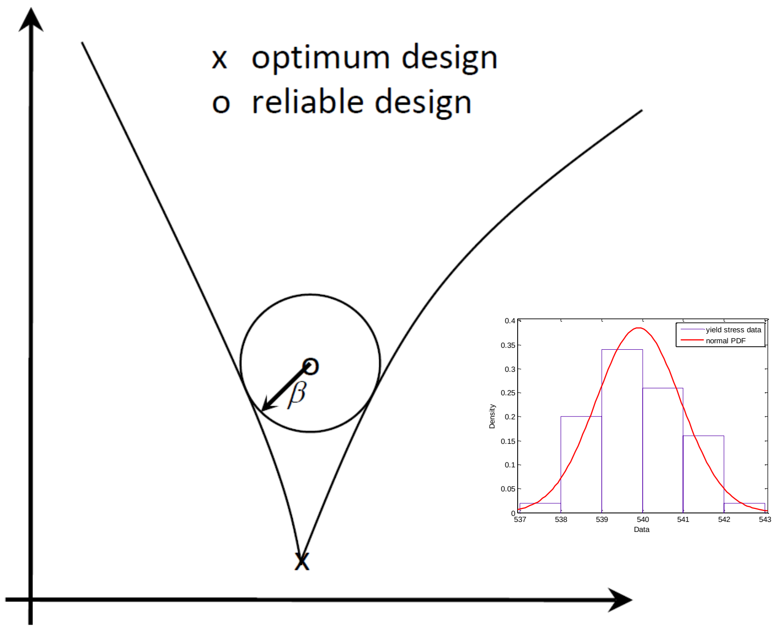


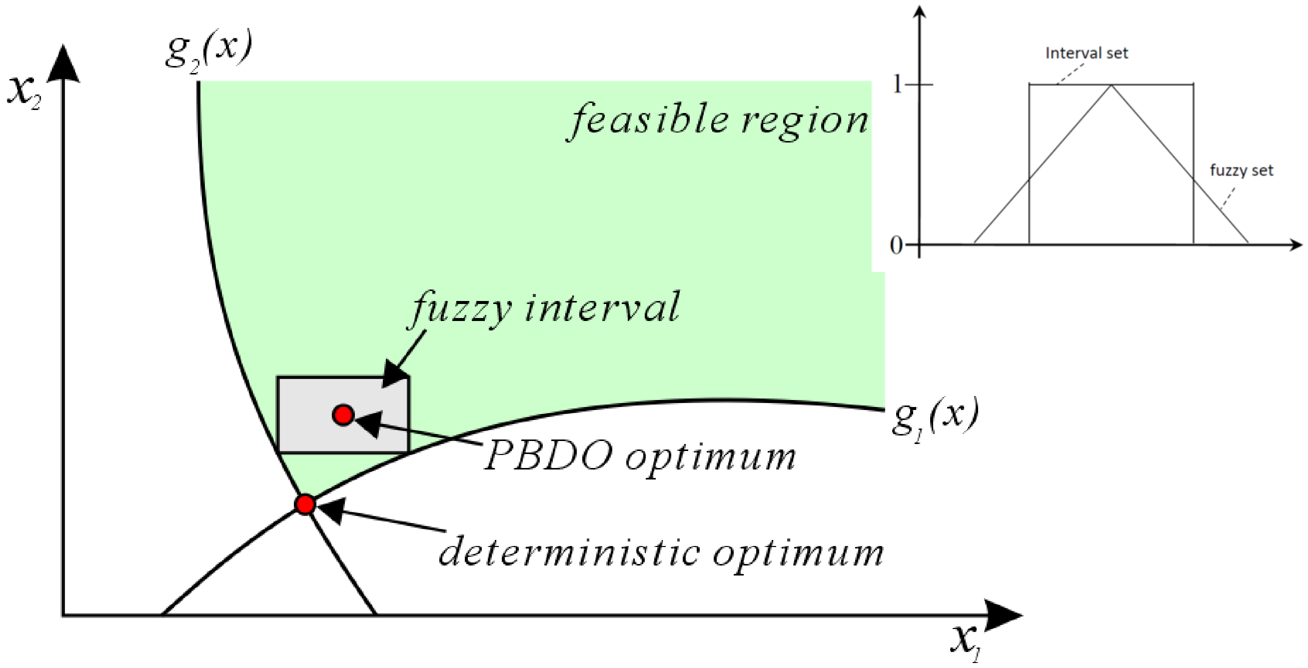
Modified constraint formulation

- Deterministic: Constraint value ≤ 0
- RBDO: Probability of (Constraint value > 0) $\leq$ Target probability

RBDO and Possibility-based Design Optimization(PBDO)

- Two similar methods that vary in the way of managing the uncertainty
  - RBDO: probability density function
  - PBDO: intervals or fuzzy numbers



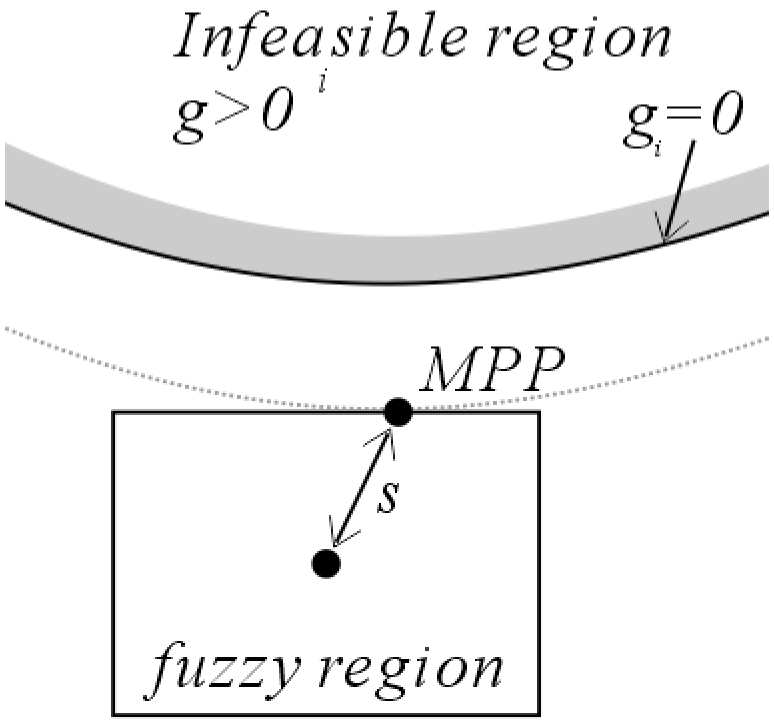


Performance Measurement Approach

- PMA is developed rather for design than reliability analysis
- Calculates the combination of uncertain variables where the value of the constraint is the worst

$$\max g(V)$$
$$s.t.\ \|V\|_\infty \le 1-\alpha_t$$

- The optimum point on this domain is identified as the most possible point (MPP)
- Usually solved via gradient methods (SQP)



PBDO Solution Strategy – Sequential Method

- Run deterministic optimization at first iteration

$$\min f(d, \bar{x}, \bar{p})$$
$$s.t.\ g_i(d, \bar{x}, \bar{p}) \le 0$$

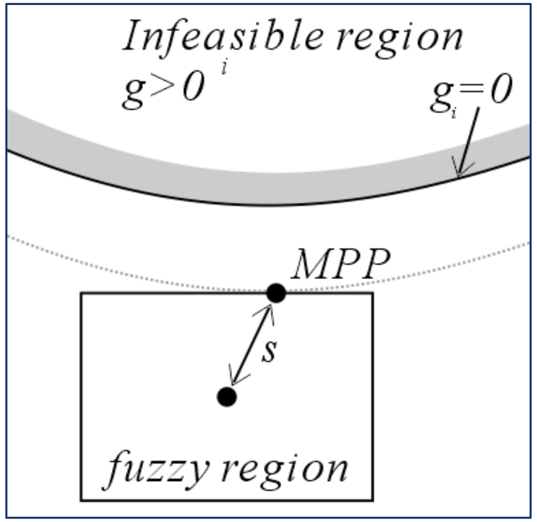
- Find x_{MPP}, p_{MPP} using PMA
- Calculate shift vector

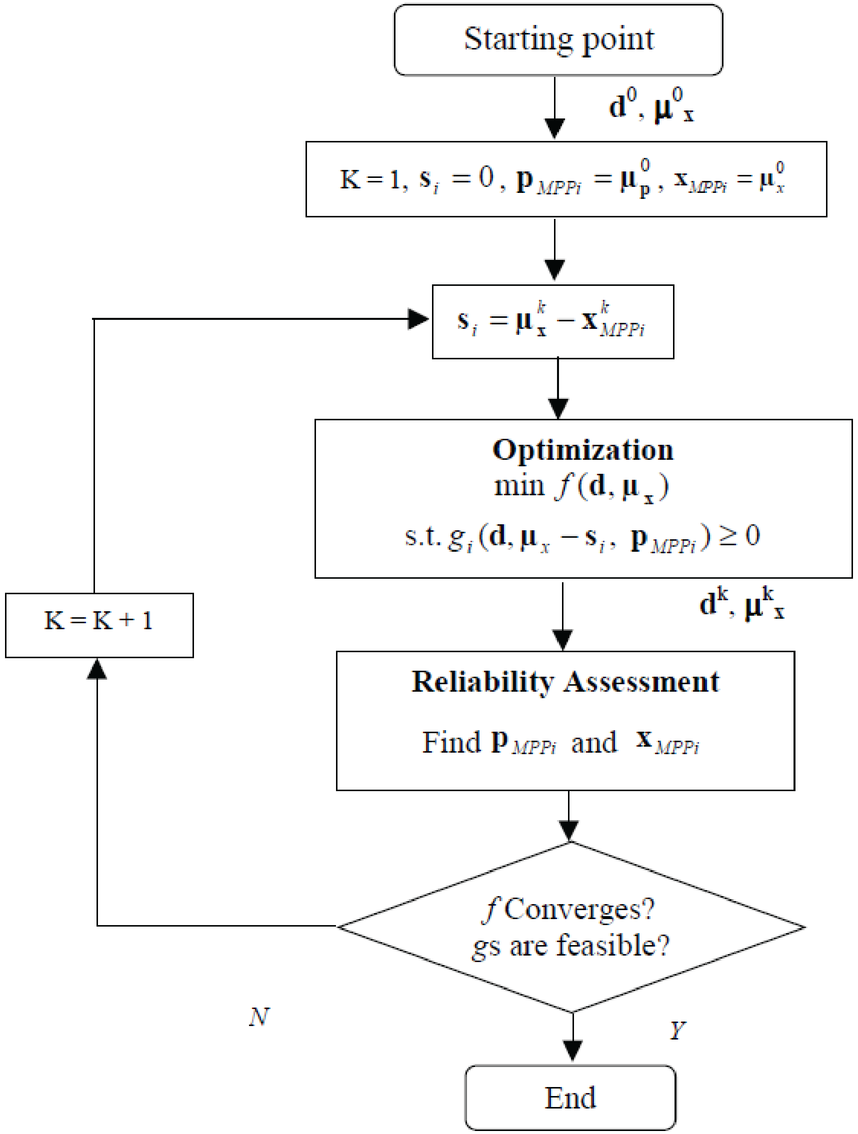
$$s_i = \bar{x} - x_{MPP_i}$$

- Run deterministic optimization

$$\min f(d, \bar{x}, \bar{p})$$
$$s.t.\ g_i(d, \bar{x} - s_i, \bar{p}) \le 0$$

- Iterate until convergence





Benchmarking The Accuracy of Analysis Software

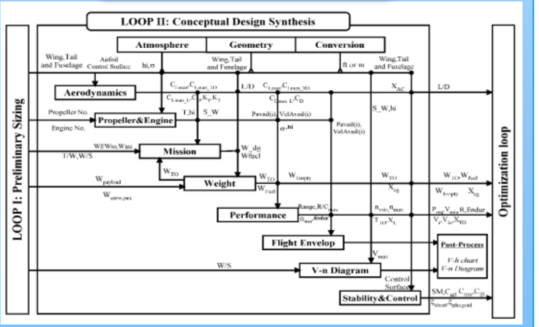
Aircraft Database

Group	Description	Group	Description
Configuration	• Wing, HT, VT geometry • Fuselage shape • Landing gear location	Propulsion	• Engine • Maximum thrust • SFC • Propeller RPM
Weight	• Empty mass • Maximum takeoff mass • Fuel mass	Performance	• Minimum speed • Maximum speed • Maximum range • Maximum endurance • Maximum rate of climb • Takeoff distance • Landing distance

Configuration

Perf. Data (Y_{db})

Analysis

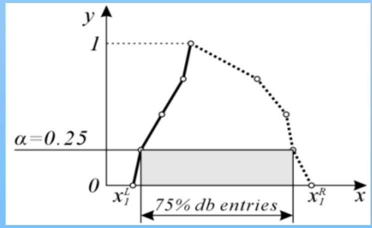


Predicted Data (Y_p)

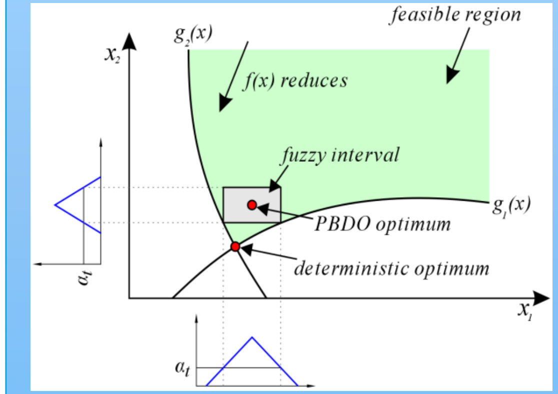
Error estimation

$$\varepsilon = \frac{Y_p}{Y_{db}} - 1$$

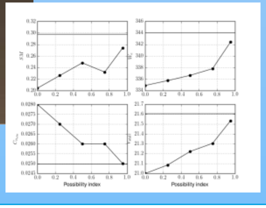
Construct Fuzzy Membership Function



PBDO

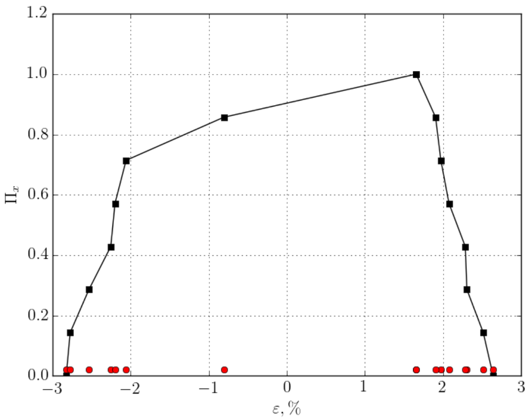


Optimum Configuration

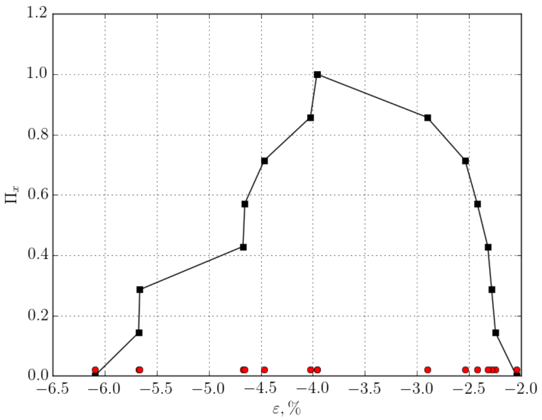


Fuzzy Membership Functions for Analysis Errors

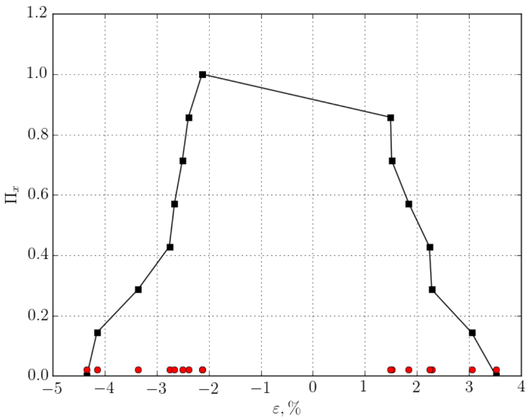
X-axis corresponds to % of DB entries used to describe the interval



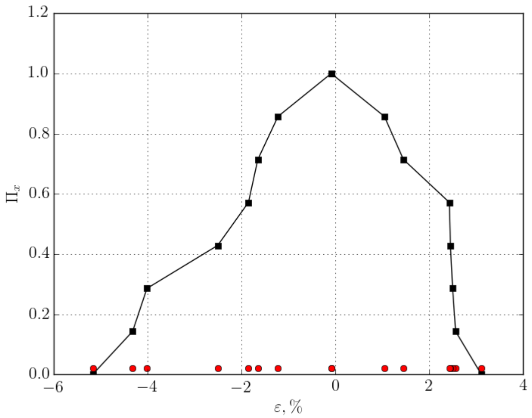
Empty weight



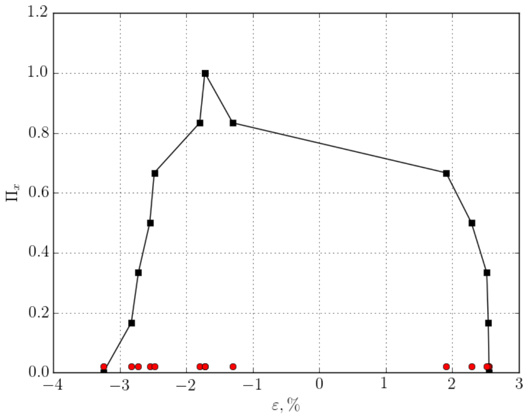
Stall speed



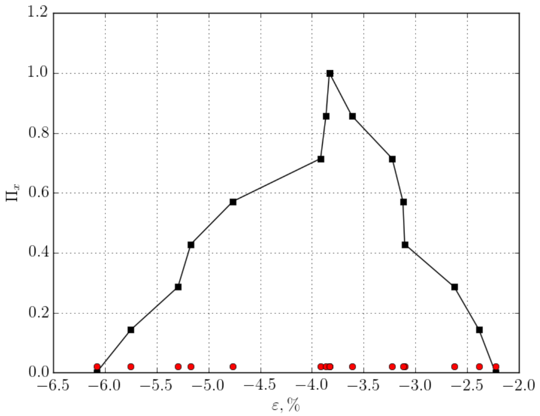
Takeoff distance



Landing distance



Rate of climb

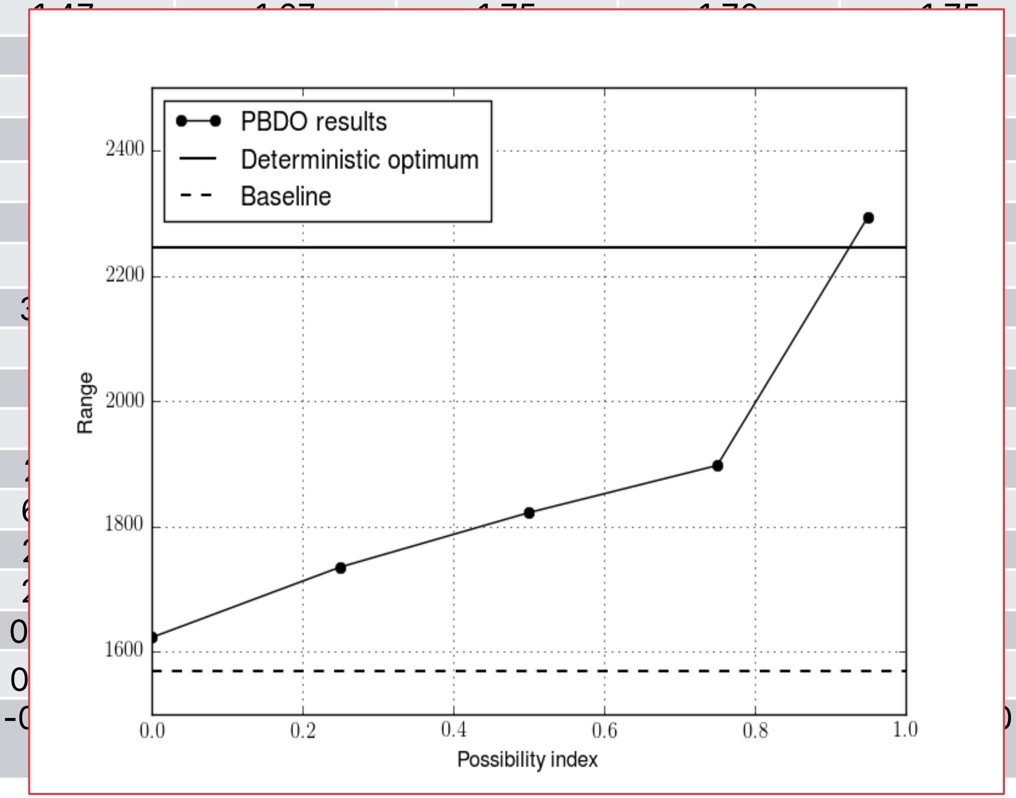


Range

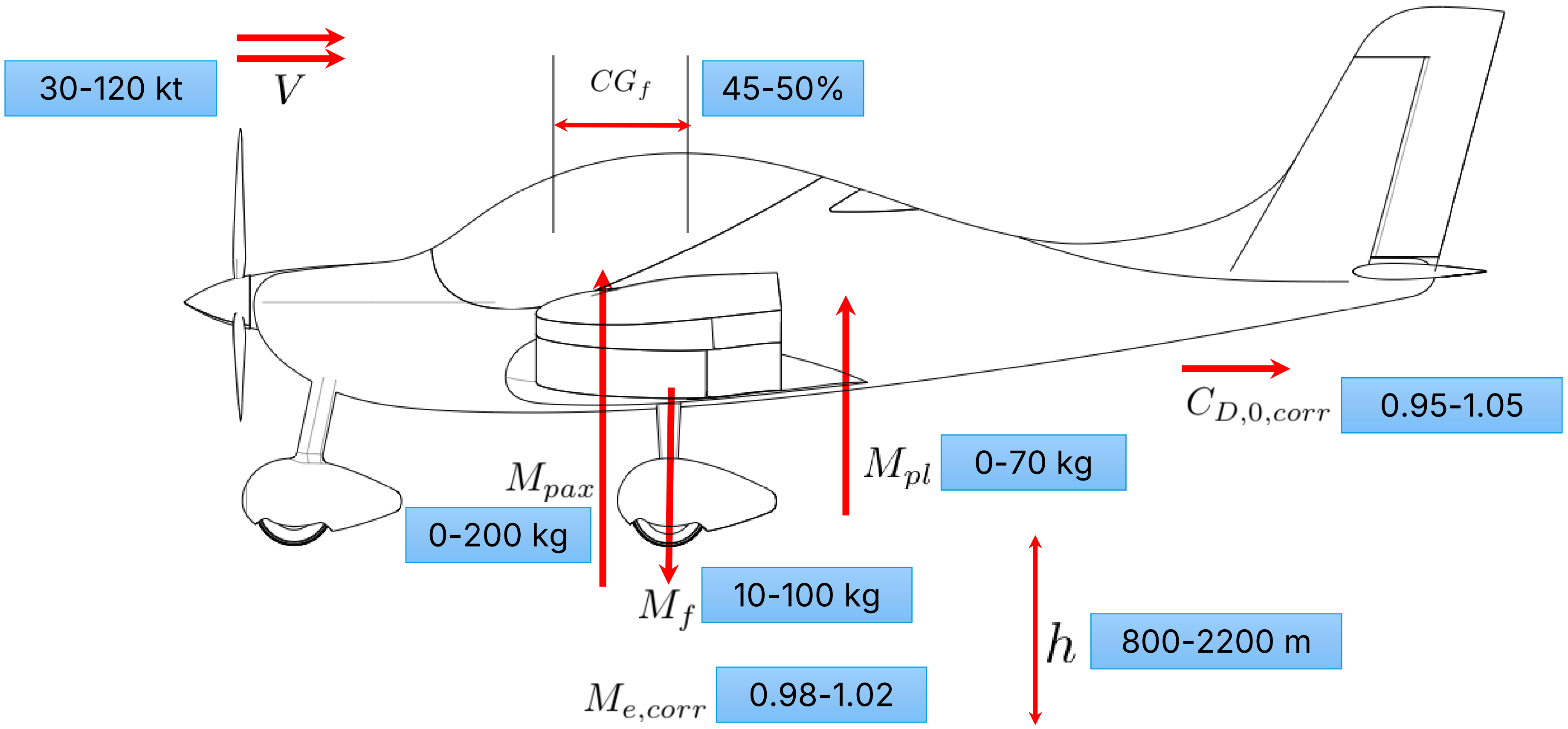
Results of Design with Analysis Error Uncertainties

Table 4: Results of Light Aircraft Design Optimization

	Variable	Unit	Baseline	Deterministic	Database Coverage				
					100%	75%	50%	25%	5%
	R	km	1570	2245	1623	1735	1822	1897	2294
Design variables	AR_w	-	7.92	9.91	7.10	7.36	7.72	8.02	9.35
	S_w	m^2	11.40	10.28	11.40	11.32	11.12	11.08	11.18
	X_w	m	1.41	1.74	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	AR_h	-	4.77	3.34	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	S_h	m^2	3.10	1.86	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	η_h	-	1.00	1.00	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	AR_v	-	1.63	2.28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	S_v	m^2	1.04	0.62	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	η_v	-	0.60	0.71	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Constraints mean value	W_e	kg	344.6	344.0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	L/D	-	5.43	6.08	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	R	km	1570	2246	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	R/C	m/s	5.23	5.36	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	V_{stall}	m/s	20.84	21.61	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	V_{max}	m/s	65.09	67.41	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	l_{TO}	m	282.0	297.1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	l_{LDG}	m	278.3	284.1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	SM	-	0.2475	0.2980	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	C_{n_β}	$1/rad$	0.0395	0.0250	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	C_{l_β}	$1/rad$	0.0236	0.0241	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]



Tail Design with Uncertain Loading and Operating Conditions



Design Formulation

- **Design Constraints**
  1. Static margin > 8%
  2. Elevator < 30 deg. on approach
  3. Elevator < 30 deg. on takeoff rotation
  4. Yaw moment coefficient due to sideslip >0.04
  5. Roll moment due to sideslip < -0.03
  6. Dutch roll frequency > 1 rad/s
  7. Dutch roll damping > 0.08
  8. Short period frequency > 2, < 10
  9. Short period damping > 0.35, < 2
  10. Stall speed < 40 kts
  11. Takeoff distance < 330 m
  12. Landing distance < 330 m
  13. Maximum speed > 120 kts

- **Design Formulation**

$$\text{minimize mass} = f(x, P)$$
$$x = [b_h, c_h, S_v, x_{mw}, b_{mw}, c_{mw}]$$
$$P = [M_{e,corr}, C_{D,0,corr}, CG_f, M_{pax}, M_{pl}, M_f, h, V]$$
$$\text{such that } g_i(x + S_i, P_{mpp,i}) \leq 0$$

- **Optimum Design Variables**

	b_h m	c_h m	S_v m^2	x_{mw} m	b_{mw} m	c_{mw} m
x_{lb}	2.00	0.40	0.20	1.00	7.00	0.8
x_{ub}	5.00	0.65	2.50	5.00	15.0	1.2
x^*_{det}	2.00	0.59	1.28	1.55	9.22	1.2
x^*_1	3.17	0.64	1.35	1.57	9.67	1.2
x^*_2	3.15	0.65	1.35	1.58	9.67	1.2
x^*_{opt}	3.13	0.65	1.35	1.56	9.67	1.2

Most Probable Points (MPP) for Constraints

The most critical combination of uncertain parameters for each constraint

Uncertain Parameters

Constraints	M_g kg	CG_f -	M_{pax} kg	M_{pl} kg	M_f kg	CG m	$C_{D,0}$ -	h m	V kt		goal	g $\bar{p}$	g p_{mpp}
h_n	585	0.50	34	70	35	2.00	0.027	1563	54	$\geq$	8%	20%	8%
$\delta_{e,a}$	597	0.40	176	0	61	1.74	0.029	3000	105	$\leq$	30°	11°	19°
$\delta_{e,r}$	596	0.40	200	0	41	1.74	0.029	3000	104	$\leq$	30°	16°	30°
C_{n_β}	602	0.50	181	68	29	1.95	0.030	3000	117	$\geq$	0.040	0.043	0.04
C_{l_β}	589	0.48	195	63	16	1.93	0.028	0	117	$\leq$	-0.03	-0.10	-0.08
S_{to}	605	0.45	107	32	79	1.85	0.030	1571		$\leq$	330m	263m	276m
V_s	605	0.46	127	40	74	1.87	0.028	1594	91	$\leq$	40kt	39kt	39kt
S_{ld}	607	0.46	128	41	74	1.88	0.025	1522		$\leq$	330m	308m	321m
V_{max}	601	0.45	108	33	80	1.85	0.030	1413		$\geq$	120kt	126kt	122kt
ζ_{dr}	598	0.50	200	68	11	1.96	0.029	3000	84	$\geq$	0.08	0.17	0.14
ω_{dr}	560	0.50	196	70	14	1.96	0.028	1530	62	$\geq$	1.00	2.28	1.85
ω_{sp}	560	0.50	111	70	99	1.97	0.025	3000	56	$\geq$	2.00	5.30	2.30
ω_{sp}	607	0.40	200	0	36	1.73	0.030	0	118	$\leq$	10.0	5.30	9.01
ζ_{sp}	599	0.40	154	0	85	1.74	0.028	3000	108	$\geq$	0.35	0.58	0.49
ζ_{sp}	576	0.50	108	70	69	1.97	0.028	0	74	$\leq$	2.00	0.58	0.71

Methods for Design under Uncertainties

RDO: Robust Design Optimization

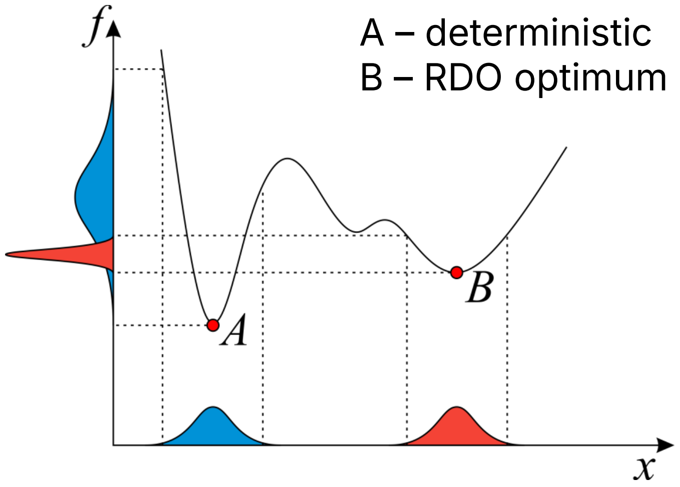
Minimizes **variation of objective** function due to uncertain parameter/variables

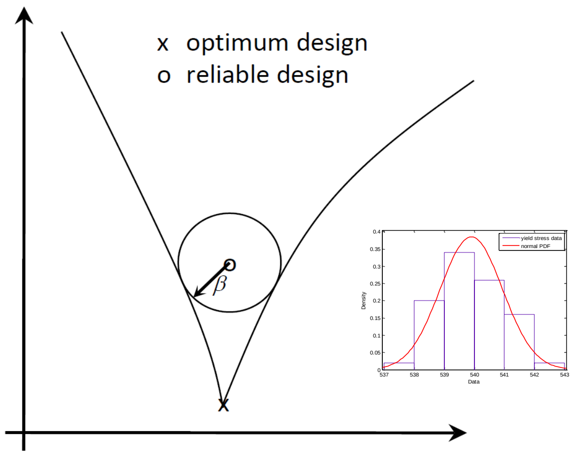
RBDO: Reliability Based Design Optimization

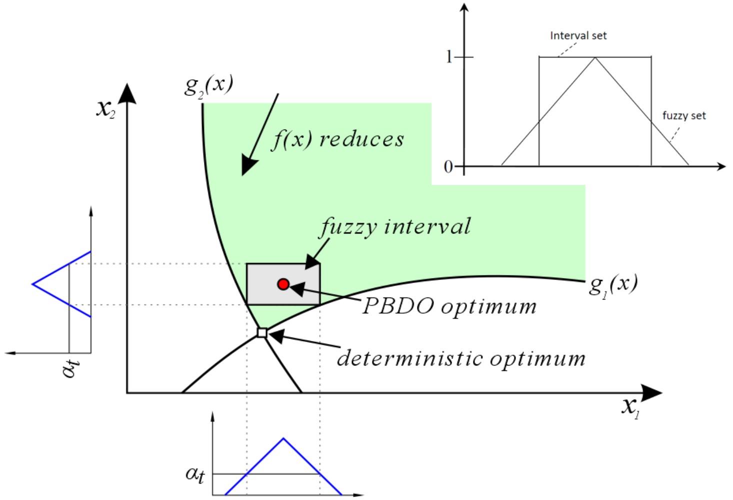
Sets constraint as: **probability** of failure less than specified value.
Uncertain parameter/variable is assumed to have **random distribution**

PBDO: Possibility Based Design Optimization

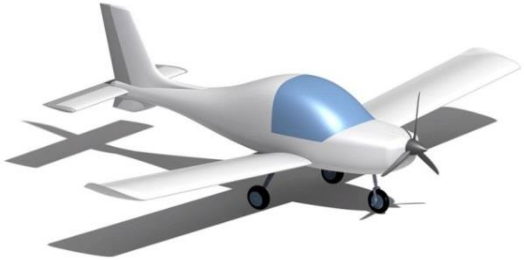
Sets constraint as: possibility of failure less than specified value. Uncertain parameter/variable is created as **interval or fuzzy number**

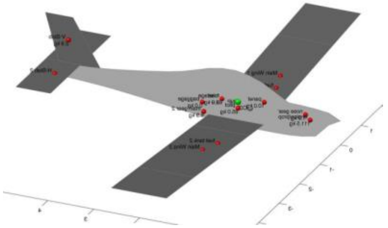


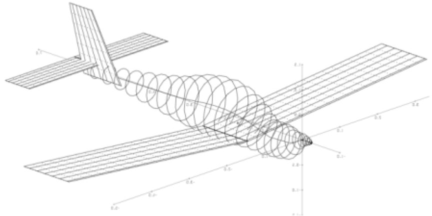


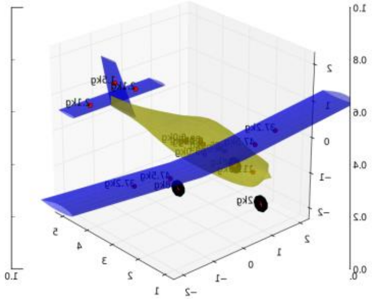


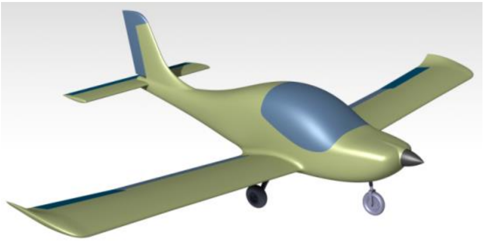
Design Iterations for The Light Aircraft Development

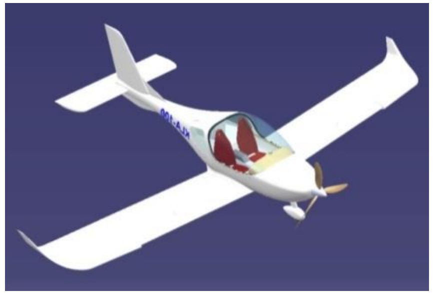


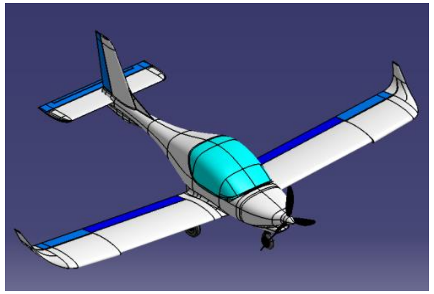












Result of Light Aircraft Development Project

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Thank You

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